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THE AMOUNT EFFECT AND MARGINAL VALUE

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The amount effect of delay discounting (by which the value of larger reward amounts is discounted by delay at a lower rate than that of smaller amounts) strictly implies that value functions (value as a function of amount) are steeper at greater delays than they are at lesser delays. That is, the amount effect and the difference in value functions at different delays are actually a single empirical finding. Amount effects of delay discounting are typically found with choice experiments. Value functions for immediate rewards have been empirically obtained by direct judgment. (Value functions for delayed rewards have not been previously obtained.) The present experiment obtained value functions for both immediate and delayed rewards by direct judgment and found them to be steeper when the rewards were delayed—hence, finding an amount effect with delay discounting.

Key words: amount effect, delay discounting, diminishing marginal value, value function

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Large amounts of money are commonly discounted less steeply by a given delay than are smaller amounts. This is the *amount effect*. For example, Stony Brook undergraduates, on average, valued \$10,000 delayed by 5 years equally to \$5,000 immediately—a 50% reduction—but they valued \$100 delayed by the same 5 years equally to \$30 immediately—a 70% reduction (Raineri & Rachlin, 1993). The amount effect is almost universally found in studies of delay discounting (Green, Myerson & McFadden, 1997). The purpose of this article is to show how the amount effect is related to the also well-established *law of diminishing marginal value* (DMV) or diminishing marginal utility. The law of DMV says that a given addition in amount adds less in value to a larger reward than it does to a smaller reward. Gabriel Cramer (1738/1954) proposed a simple power-function version of this law which for convenience we will assume to apply:

$$V = cA^n \tag{1}$$

where V is the value (or utility) of a reward of amount A, c is a scaling constant, and n is a fractional exponent ($0 \leq n \leq 1$). The marginal value function is the derivative of the value function. Cramer suggested that $n = .5$ (square root). To illustrate how marginal value diminishes, let us suppose $n = .5$. If $c = 1$ and $A_1 = 9$, then $V_1 = 3$ units of value. If $A_2 = 64$ then $V_2 = 8$ units of value. Suppose now we add 7 to both A_1 and A_2. Then $A_{1'} = 16$ and $A_{2'} = 71$. Now $V_{1'} = 4$, a gain of 1 unit, and $V_{2'} = 8.4$, a gain of only 0.4 units. This is generally true for Equation 1 with $0 < n < 1$: The larger the original amount, the less the increase in value by the addition of a fixed sum.

The present application of DMV to delay discounting supposes that when a reward is delayed it is not the amount or delay of that reward that is subject to DMV but its delay-discounted value. A commonly obtained delay discount function is the hyperbolic function of Equation 2 (Mazur, 2001):

$$a = V = \frac{A}{1 + kD} \tag{2}$$

where D is the delay of a reward of amount A, a is the amount of an immediate reward equal in value (V) to the larger but delayed reward, and k is a constant. Note that k multiplies the effect of D. As k increases, delay discounting becomes steeper. Suppose we assume that both the undiscounted (left) and delay-discounted (right) sides of Equation 2 are subject to the law of DMV:

$$c' a^m = v = c\left(\frac{A}{1 + kD}\right)^n \tag{2a}$$

If the value (v) of an immediate reward (a) were the same function of amount as the value (V) of

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a delayed reward $[A/(1+kD)]$ then in Equation 2a $c' = c$, $m = n$, and Equation 2a reverts to Equation 2. Equation 2 does not predict an amount effect. An amount effect (large rewards discounted less than smaller rewards by the same delay) implies that the equivalent immediate amount, normalized by the larger delayed amount (a/A), varies directly with A. This is clearly not the case for Equation 2 where a/A is independent of A. For this reason most theorists reject DMV as an explanation of the amount effect (Lowenstein & Prelec, 1992).

But suppose that the value function is different for immediate and delayed rewards ($m \neq n$).[1] Then:

$$a = \frac{A^{\frac{n}{m}}}{(1+kD)^{\frac{n}{m}}} \tag{3}$$

Because $A^{n/m} = A{*}A^{(n/m)-1}$:

$$\frac{a}{A} = \frac{A^{\frac{n}{m}-1}}{(1+kD)^{\frac{n}{m}}} \tag{3a}$$

Now, normalized equivalent immediate amount may vary directly with the amount of the delayed reward. For this to be the case (to obtain an amount effect), the exponent of A on the right side of Equation 3a must be positive. That is, $n > m$; the value function exponent for delayed rewards must be greater than that for immediate rewards. Because psychophysical scaling of variables such as amount and delay typically involves fractional exponents, $1 > n > m$. That is, value functions for delayed rewards are more nearly linear than those for immediate rewards.[2]

This difference in value functions for immediate and delayed rewards is not an empirical prediction—it is a necessary consequence of the amount effect. Because the amount effect is reliably found in delay discounting, the difference in value functions for immediate and delayed rewards must also be found. Figure 1 shows the necessary relations between the two effects as four points on two delay discounting functions (left) and the same four points on two value functions (right). The only difference is the way they are plotted. Then what, the reader may ask, is the point of doing this experiment? Note that the most direct test of Equation 2a would be a choice experiment wherein immediate rewards are equated in subjective value to larger–later rewards. Indeed, delay discount functions are usually obtained using such choice procedures (Rachlin, Raineri & Cross, 1991). Value functions, on the other hand, are typically inferred from economic behavior of individuals or groups and rarely determined directly. Where they are directly obtained (e.g., Galanter & Pliner, 1974) they are determined by direct judgment of value of immediate rewards. To our knowledge, value functions of delayed rewards have not been directly obtained before the present experiment.

Another difference between this experiment and delay discounting experiments is that the direct judgments in this experiment were of degree of happiness participants would (hypothetically) feel on receiving the promise of the reward whereas delay discounting experiments are typically based on direct choices between real or hypothetical rewards. The present experiment therefore may be viewed as a check on the ability of judgments of this kind to predict choices and to show that the amount effect is not an artifact of the choice procedure commonly used to obtain delay discount functions.

In obtaining value functions to test these predictions it is important that immediate and delayed value functions be obtained independently. To have participants choose between immediate and delayed rewards would be to repeat the discount experiments, the results of which we are trying to duplicate. Normally, a function relating a psychological variable (such as value) to a physical variable (such as amount) would be obtained by the method of magnitude estimation, in which participants match numbers to physical variables (Stevens, 1966). In the present experiment, however, the physical variables are already numbers (amounts of money). The task of matching numbers of one kind to numbers of another kind would be unnecessarily confusing for participants. Instead we used a magnitude production task

[1] For simplicity we assume $c = c'$.

[2] A psychophysical analysis of discounting pursued by Green, Myerson, and associates exponentiates the denominator of the right side of Equation 3a by a fraction (s). The fractional exponent implies a diminishing marginal effect of delay. In a series of experiments they have found that this model accounts for extant data much better than does Equation 2, and that s remains constant over a very wide range of conditions (e.g., Green et al., 2013) and thus may reflect subjective evaluation of delay. However, this model does not explain the amount effect (nor was it intended to do so).

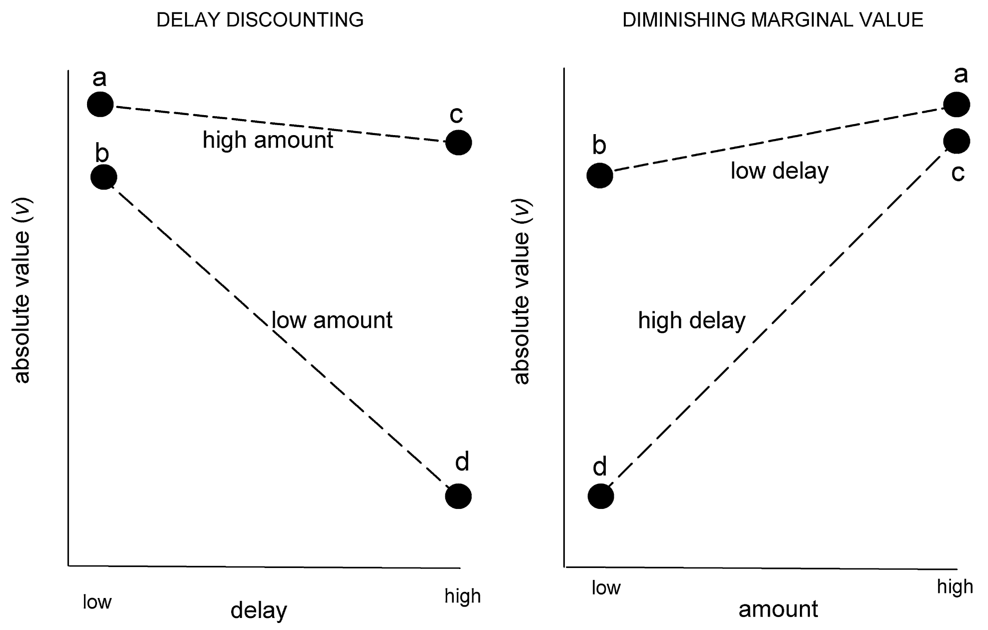

Fig. 1. Left: Four points arranged to show an amount effect with delay discounting. The degree of discounting of the high amount is less than that of the low amount. Right: The same four points arranged to show differing steepness of marginal value functions at different delays. Note, the marginal value function for the high delay is steeper than that for the low delay. Corresponding points on the left and right sets of axes are in line (at the same absolute value). Because the angles of the drawn lines may create the illusion that corresponding points are out of line, the reader may want to confirm their alignment with a ruler.

(Stevens, 1966) in which participants adjusted the length of a line to match the value of a delayed reward.

Method

Participants

Eighty-seven Stony Brook University students received course credit through Stony Brook's SONA system for participation. Sixty-four (74%) were female. The median age was 19. Sixty-seven (77%) identified themselves as native English speakers, and all but two of the remainder described themselves as fluent English speakers.

Procedure

Participants were asked to imagine that they were eligible for a monetary prize to be received after a given delay. (See Appendix for instructions.) For each of several amount–delay pairs, participants held down the hyphen key until a line or multiple lines of hyphens represented how happy they would be to receive the prize; greater lengths indicated greater happiness. This is a standard cross-modality method for estimating psychological value (Galanter & Pliner, 1974; Stevens, 1975). To avoid scale compression at the extremes, if participants felt that the line would be longer or shorter than practically possible to draw, they were permitted to type a phrase such as "5 millimeters" or "half a mile" to indicate how happy they would be to receive the prize.

Nine amount–delay pairs were judged; there were three possible amounts of money ($10; $10,000; $1 million), and three possible delays (immediately; 1 year; 5 years). For each participant, amount–delay pairs were presented in random order except that the intermediate pair ($10,000 in 1 year) was always presented first. By putting the same item first, we hoped to reduce heterogeneity in how participants scaled their responses.

Participants completed the study over the Internet on their own time. Some of the participants also performed a probability judgment task, but since they all were not tested with probability and because the order of delay and

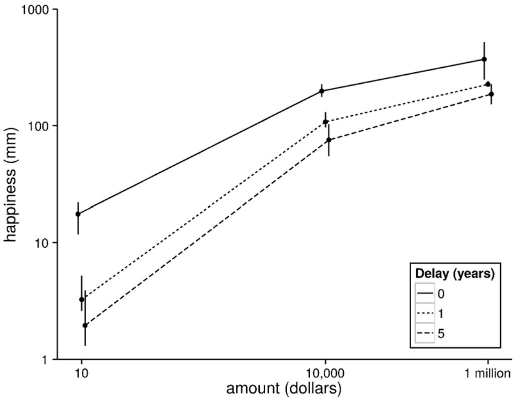

Fig. 2. Median line-length for each amount–delay pair. Axes are logarithmic. Points are shifted horizontally to keep the error bars from overlapping. The error bars show 40th and 60th percentiles.

Table 1

Median slopes of segments of individual-participant DMV functions.

Delay (years)	*\$10 -\$10,000*	*\$10,000 - \$1 million*
0	.298	.199
1	.434	.162
5	.401	.194

probability tasks was not randomized or counterbalanced (probability was always tested after delay), only the delay task results will be presented.

Results

All line lengths were standardized to millimeters. Written descriptions such as "3 feet" were converted to millimeters. Single hyphens were regarded as 1.3 mm long.[3] Responses that did not specify a length, such as "very strong", were excluded from analysis. One subject described several infinite lengths, so all her responses were excluded. After these exclusions, 783 amount–delay pairs remained for analysis; 91% were drawn lines, and the rest were numerical distances.

Figure 2 shows the median lengths drawn by participants for each amount and delay. Naturally, participants expressed more happiness for greater amounts of money and sooner payouts. Table 1 shows the median of individual-participant slopes of the six line segments of the delay log-log plots (see Fig. 2). All slopes are less than 1.0 indicating diminishing marginal value. In all cases, the median slopes of the segments from \$10 to \$10,000 were steeper than those from \$10,000 to \$1 million, evincing a downward bend in the overall function.

In a prior experiment (Galanter & Pliner, 1974), participants indicated happiness for receipt of various monetary amounts by adjusting the intensity of a tone, and assigned numerical values to various intensities. From these functions the authors were able to obtain a relation of happiness (as numerical value) to monetary amount. They found a linear relationship with a slope of 0.45 between happiness and money on a log–log plot [$\text{happiness} = k(\$)^{0.45}$] close to the square-root marginal value function suggested by Cramer (1738/1954). In the present experiment, the overall slope of the regression line of the 0-delay DMV function in Figure 2 is 0.27. The slope of the initial segment is 0.34; that of the second segment is 0.14. These low slopes indicate compression of judgments, especially at the upper end of the functions relative to the linear functions obtained over the (much narrower) range of monetary amounts scaled by Galanter and Pliner.

The main object of this experiment was to determine whether the DMV functions for lower amounts were steeper overall than the DMV functions for higher amounts. The median values of Figure 2 with delayed rewards do show a difference in slope in the direction predicted. To statistically test the difference in slopes of the DMVs of Figure 2, for each subject, we calculated the ratio:

$$\frac{\log v_0(\$10) - \log v_1(\$10)}{\log v_0(\$1\,\text{million}) - \log v_1(\$1\,\text{million})} \qquad (4)$$

In terms of Figure 1, the test would be [log(a) – log(b)]/[log(c) – log(d)]. The antilog of the difference between two logs (as in the equation) equals the ratio of the numbers. If the

[3] The actual on-screen length of a hyphen depended on the resolution of the subject's screen and whether the subject was using magnification. We did not measure these factors. However, our choice of 1.3 mm is unlikely to influence our analysis because the measured line-lengths varied proportionally within participants and the variance in the written line lengths dwarfed that of the reasonably expected screen sizes.

slope of the high-delay value function differed from that of the slope of the low-delay value function (as shown in the right section of Fig. 1) then the above expression would differ from 1.0. This is what was tested. First we compared the functions where $v_0(x)$ is the subject's response for x immediately, and $v_1(x)$ is the subject's response for x with delay 1 year. We included only participants who indicated greater happiness for greater amounts within the two delays considered [i.e, with $v_0(\$1 \text{ million}) > v_0(\$10)$ and $v_1(\$1 \text{ million}) > v_1(\$10)$] and who discounted over the appropriate delay change [i.e., with $v_0(\$10) > v_1(\$10)$ and $v_0(\$1 \text{ million}) > v_1(\$1 \text{ million})$]. Application of these criteria left 61 participants. We tested the null hypothesis that the mean of these ratios was 1. To reduce skew, we log-transformed the ratios (and also the null mean) before conducting single-sample *t*-tests. We found that participants were significantly less sensitive to delay when the amount was larger, $t(60) = 3.87$, $p < .001$. Then we compared the functions where $v_0(x)$ is the participant's response for x immediately, and $v_1(x)$ is the participant's response for x with delay 5 years. Re-application of the aforementioned inclusion criteria for these new responses left 68 participants. Again, participants were significantly less sensitive to delay when the amount was larger, $t(67) = 3.88$, $p < .001$.

Discussion and Conclusions

The present experiment obtained marginal value functions for both immediate and delayed rewards by direct judgment and found them to be steeper when the rewards were delayed; hence, finding an amount effect with delay discounting. Note that the value functions of Figure 2 would eventually meet as amount increased indefinitely; to put the same result another way, delay discount functions would eventually become flat.

Note also that the difference of slopes of the value functions of Figure 2 occurs almost wholly between the zero-delay and 1-year functions. The 1-year and 5-year functions have nearly the same slope. This finding raises the question whether immediate and delayed amounts are differently scaled, or whether scaling (i.e., the exponent in the value function) changes continuously with delay. The present results do not provide a direct answer to this question. Direct measurement of the value function for delays between 0 and 1 year might resolve this question.

Another candidate for an explanatory mechanism is Killeen's (2015) arithmetic theory of discounting. In that theory delay subtracts from the value of a reward. A fixed delay would subtract the same amount from large and small rewards. For high fixed delays, therefore, the fixed amount subtracted would become more and more of a factor relative to amount in determining overall value—thus, overall value would be less sensitive to amount for high than low fixed delays as was found in this experiment.

It is worth emphasizing what this experiment does not show. It does not show that the amount effect is explained by or is caused by differences in value functions for different delays. The amount effect in delay discounting is typically obtained with choice data. The present experiment shows that the amount/DMV effect may be obtained with absolute judgments as well as with choices. Nevertheless, the experiment may have heuristic value when it comes to explaining the amount effect. Instead of asking why large amounts are discounted less by delay than are small amounts, one can ask why the value of delayed rewards grows more steeply (or diminishes less) with amount than does the value of immediate (or less-delayed) rewards. These explanations are essentially two sides of the same coin (two aspects of the same behavioral pattern). Framing the results the latter way emphasizes factors that might reasonably underlie both effects such as differences in ability to plan for consumption of delayed versus immediate rewards or differences in consumption patterns of those rewards (Rachlin, Raineri & Cross, 1991).

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APPENDIX

Instructions:

This is an experiment in imagination (of a pleasant kind). At the top of each of the following pages are an amount of money and a period of time. We ask you to imagine that you just received news that you won a prize of that amount to be received after that time. Then, below the stated amount and time we would like you to draw a horizontal line proportional to how happy you would feel now to be awarded the prize. For the first amount and time, just draw a line you feel comfortable with. After that, if you would feel twice as happy as before then draw a line twice as long. If you would feel half as happy then draw a line half as long, and so forth.

Draw your line by holding the hyphen key (−) on your keyboard, between the 0 and = keys. If your input is wider than the text box, it will wrap to the next line. If you wish to express an extremely short or extremely long length, rather than typing hyphens, type a number and unit such as “5 millimeters” or “3 miles”.

Of course there are no right or wrong answers. We just want to know how you would feel if you had gotten the prize. Please take your time and imagine how you feel then draw the line. It’s your feelings we’re interested in.

Thank you!